# Note on the Production of Pairs by Charged Particles

J. R. OPPENHEIMER, *Department of Physics, University of California*

(Received December 3, 1934)

We compute the internal conversion by pair-production of the radiation emitted in the impact of charged particles. When the energy available for pair-production is large compared to $mc^2$, this simple method gives a valid estimate of pair-production by the impact. For electrons of very high energy the probability of pair-production increases as $[\ln (E/mc^2)]^2$.

THE production of pairs by charged particles of high energy has been considered by Furry and Carlson,[1] and by Heitler and Nordheim.[2] In both of these researches the probability of pair production is calculated by an application of the systematic Born-Møller perturbation theory for relativistic impacts. We want here to consider a rather simpler method of treating the problem for the case that the energy available for pair production is very large compared to $mc^2$.

Consider first pair production by the impacts of a heavy charged particle on a nucleus, where we may take the velocity of the particle $v \ll c$, and where, in order that large energies may be available for pair production, we shall suppose that $\mu v^2 \gg mc^2$, where $\mu = M_1M_2/(M_1+M_2)$ is the reduced mass of particle and nucleus. For this case the impact of the two heavy particles and the electromagnetic field produced by the impact can be computed purely classically, and we have to study the production of pairs by this field. For the components of this field of frequency $\nu \gg \nu_0 = mc^2/h$, the wavelength is small compared to the region, of order $\hbar/mc$, in which pair production is important, so that the radiation field of the impact may, despite $v \ll c$, be supposed in this limit to contribute essentially to pair production.

It is easy to calculate the effect of this radiation field, if we neglect altogether the diffraction zone field of the charges. To do this we may use the classical expression for the intensity of the continuous x-ray spectrum. If mass and charge of the two particles are $M_1$, $Z_1e$; $M_2$, $Z_2e$, respectively, then for unit incident flux of impacting particles the intensity per unit frequency of emitted radiation is given by the well-known formula of Kramers

$$I_\nu = 32\pi^2 3^{-\frac{3}{2}} \frac{e^6 Z_1^2 Z_2^2 \zeta^2}{\mu^2 v^2 c^3}, \qquad (1)$$

for $\qquad \nu \lessgtr \bar{\nu} = \mu v^2/2h,$

where $\zeta = (M_1Z_2 - M_2Z_1)/(M_1+M_2)$ is the effective charge. This radiation is dipole radiation, for which the internal conversion coefficient by production of pairs is known.[3] For $\nu \gg \nu_0$, it is given with good approximation by

$$(2\alpha/3\pi)[\ln (2\nu/\nu_0) - 3/5]; \quad \alpha = e^2/\hbar c. \qquad (2)$$

If we take from (1) the number of quanta emitted per unit frequency per unit time, we get, using (2), for the cross section for pair production

$$\sigma = 2Q \int^{\bar{\nu}} (d\nu/\nu)[\ln (2\nu/\nu_0) - 3/5];$$

with
$$Q = 16 \cdot 3^{-5/2} \alpha^2 \frac{e^4 Z_1^2 Z_2^2 \zeta^2}{\mu^2 v^2 c^2}, \qquad (3)$$

where the lower limit of the integral over $\nu$ is of order $\nu_0$. For $\bar{\nu} \gg \nu_0$, we thus get

$$\sigma = Q[\ln (\bar{\nu}/\nu_0)]^2. \qquad (4)$$

The estimate which Heitler and Nordheim give for this case is just

$$\sigma \sim Q. \qquad (5)$$

This result they have obtained by applying Born's approximation to the impact of the two heavy particles and the calculation of their field. Although the validity of this method of

[1] W. H. Furry and J. F. Carlson, Phys. Rev. **44**, 237 (1933).

[2] W. Heitler and L. Nordheim, J. de Physique **5**, 449 (1934).

[3] L. Nedelsky and J. R. Oppenheimer, Phys. Rev. **44**, 948 (1933).

treating an impact with $Z_1Z_2 \gg 1$, $v \ll c$, can hardly be established, it probably gives correct results for the special case of the Coulomb field. On the other hand, the fields calculated by Heitler and Nordheim are just the diffraction zone fields which we have neglected; they, in turn, have neglected the radiation field entirely. The fact that for large $\bar{\nu}/\nu_0$, (4) is larger in order of magnitude than (5), may be regarded as a justification for our neglect of the diffraction zone fields, and of the validity, for $\bar{\nu} \gg \nu_0$, of (4). For $\bar{\nu} \sim \nu_0$, on the other hand, (4) and (5) give contributions of the same order of magnitude, so that both fields must be considered.

This same method can be applied to pair production by electrons of energy $E \gg mc^2$. Here we have again to limit ourselves to impacts in which the reaction of the pairs on the impacting electron can be neglected. This restriction, which is also involved in the calculations of Carlson and Furry, means that the energy of the pair shall be only a small part of $E$. As was observed by these authors, and as follows also from our calculations, this restriction does not, for large $E$, affect the order of magnitude of the results. The neglect of the diffraction zone parts of the field of the particles is here even less open to question than in the case of heavy charges.

We have now, however, to use in place of (1) the expression for the continuous radiation from high energy electrons in a Coulomb field, for which Bethe and Heitler[4] give, for $h\nu \ll E$, $h\nu \gg mc^2$,

$$I_\nu = \frac{32\pi Z^2 e^6}{3m^2c^5} \ln (\bar{\nu}^2/\nu\nu_0)$$

$$\text{with} \qquad \bar{\nu} = E/h. \tag{6}$$

This radiation is no longer dipole radiation; but for $\nu \gg \nu_0$, the dominant term in the internal conversion coefficient for multipole radiation[5] is again given by

$$(2\alpha/3\pi) \ln (\nu/\nu_0) \tag{7}$$

We thus find, for the cross section for pair production

$$\sigma = (32/9\pi)\alpha^2 Z^2 (e^4/m^2c^4) \int^{\bar{\nu}} (d\nu/\nu) \ln (\nu/\nu_0)[2 \ln (\bar{\nu}/\nu_0) + \ln (\nu_0/\nu)]$$

$$\backsimeq (64/27\pi)\alpha^2 Z^2 (e^4/m^2c^4)[\ln (\bar{\nu}/\nu_0)]^3. \tag{8}$$

The calculation of internal conversion, and of the radiation by heavy ions, will be unaffected by the screening of the nuclear field by atomic electrons. But as Bethe and Heitler[4] have shown, for $E > mc^2/\alpha Z^{\frac{1}{3}}$, (6) must be modified. Instead we have, again for $h\nu \ll E$,

$$I_\nu = (32\pi Z^2 e^6/3m^2c^5) \ln (199/Z^{\frac{1}{3}}), \tag{9}$$

which leads to

$$\sigma \backsimeq (16/9\pi)\alpha^2 Z^2 (e^4/m^2c^4) \ln (199/Z^{\frac{1}{3}})[\ln (\bar{\nu}/\nu_0)]^2. \tag{10}$$

Formulae (8) and (10) show that the probability of pair production by a very high energy electron differs from that for a gamma-ray of the same energy by a factor of the order $\alpha[\ln (\bar{\nu}/\nu_0)]^2$. This is in complete agreement with the conclusions of Furry and Carlson. For energies of $5 \times 10^9$ volts, (10) gives for the ratio of these probabilities about 10 percent.

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[4] H. Bethe and W. Heitler, Proc. Roy. Soc. **A146**, 83 (1934).

[5] For $\nu \gg \nu_0$, radiation corresponding to multipoles of very high order will be present; for these the use of the asymptotic value (7) for the internal conversion coefficient may be questioned. The true value of the internal conversion coefficient will be smaller than (7). This correction will not affect the dominant term of the result.